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MATH 2312
Pre-Calculus II
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8.3 Trigonometric Equations

values that satisfy an equation called solutions

ex: determine if $\theta = \frac{\pi}{4}$ is a solution to the equation.

$2\sin\theta - 1 = 0$

$2\sin\left(\frac{\pi}{4}\right) - 1 = 0$

$2\left(\frac{\sqrt{2}}{2}\right) - 1 = 0$

$\sqrt{2} - 1 = 0$

$\sqrt{2} \neq 1$

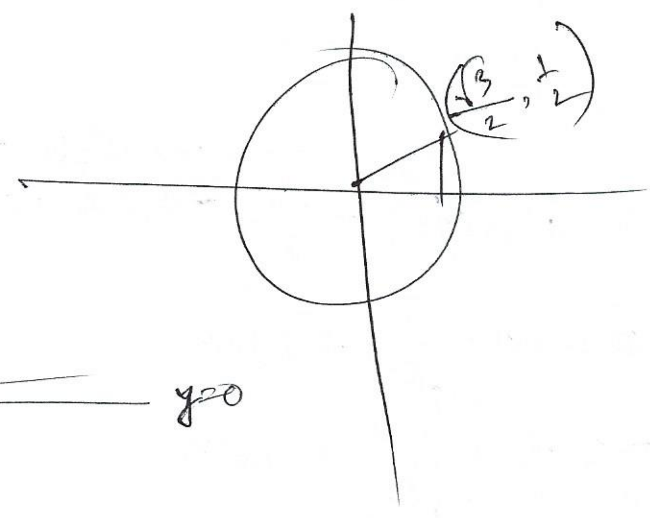
NO, $\frac{\pi}{4}$ not a solution

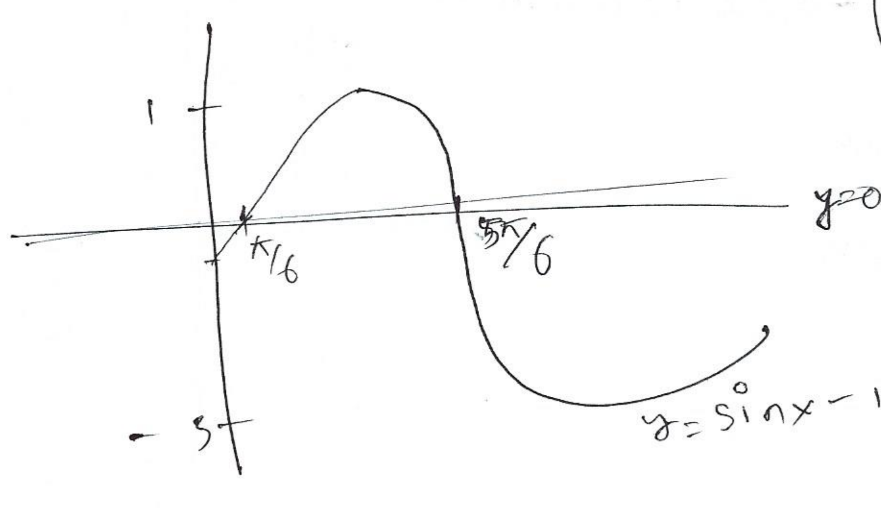
$\theta = \frac{\pi}{6}$ is a solution

$2\sin\left(\frac{\pi}{6}\right) - 1 = 0$

$2\left(\frac{1}{2}\right) - 1 = 0$

$1 - 1 = 0$ ✓





$\theta = \frac{\pi}{6} + 2k\pi$ if $k = 1$

$\theta = \frac{\pi}{6} + 2\pi$

$= \frac{13\pi}{6}$

$\theta = \frac{5\pi}{6} + 2k\pi$

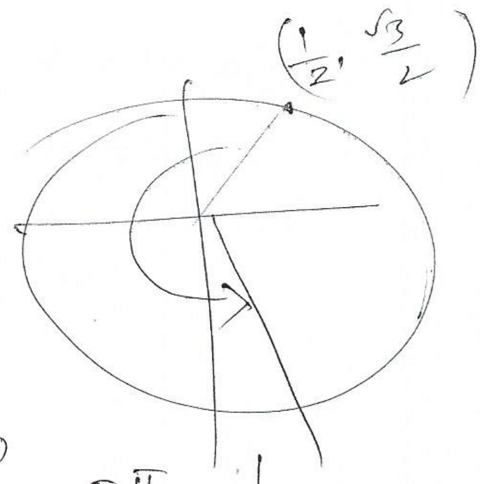
$= \frac{17\pi}{6}$.

$\cos\theta = \frac{1}{2}$

solve, give ~~for~~ general form list eight solutions

$[0, 2\pi)$

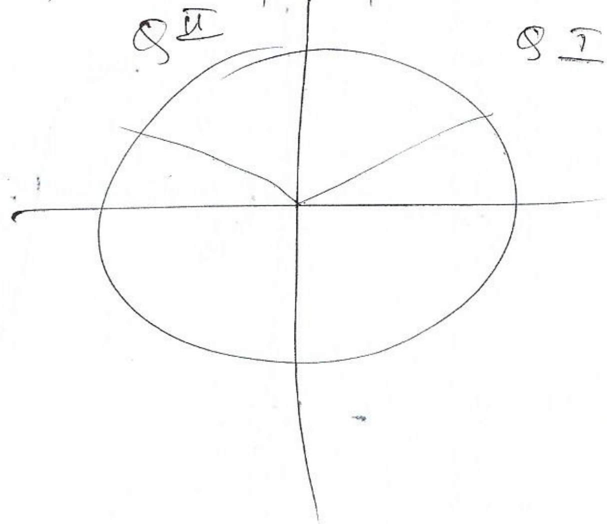
$T = 2\pi$



→ multiple of θ

→ Use general form

$\sin(2\theta) = \frac{1}{2}$

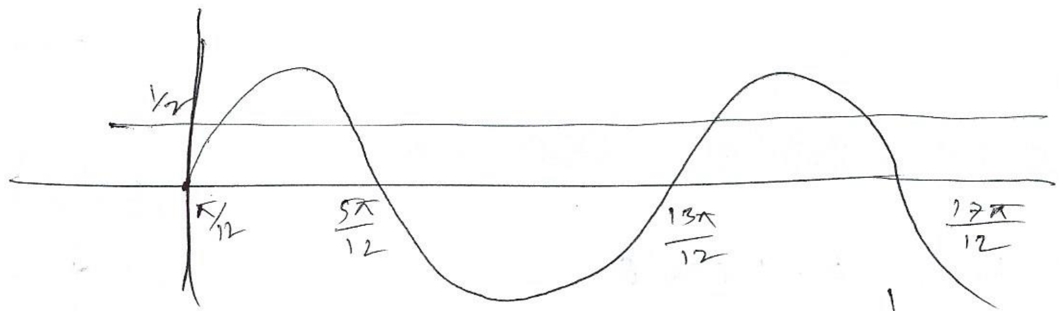


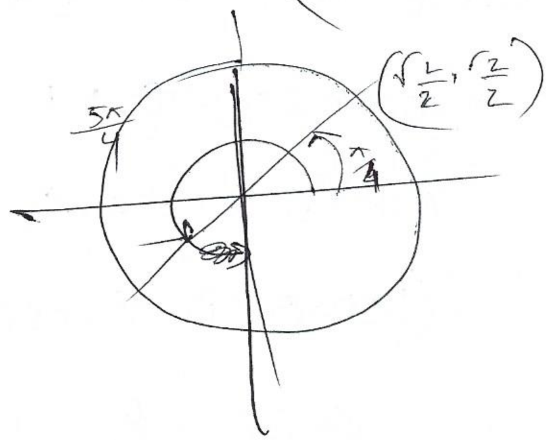
$2\theta = \alpha_1 = \frac{\pi}{6} + 2k\pi$

$2\theta = \alpha_1 = \frac{5\pi}{6} + 2k\pi$

$\theta = \frac{\pi}{12} + k\pi$

$\theta = \frac{5}{12} + k\pi$





$\tan\left(\theta - \frac{\pi}{2}\right) = 1$

$\theta - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$

$\theta - \frac{\pi}{2} = k\pi + \frac{5\pi}{4}$

$\theta = \frac{3\pi}{4} + k\pi$

$\theta = \frac{7\pi}{4} + k\pi$

Quadratic in form

Solve $2\sin^2\theta - 3\sin\theta + 1 = 0 \qquad 0 \le \theta < 2\pi$

$2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$

$2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta = 1$ $\qquad (0, 1)$

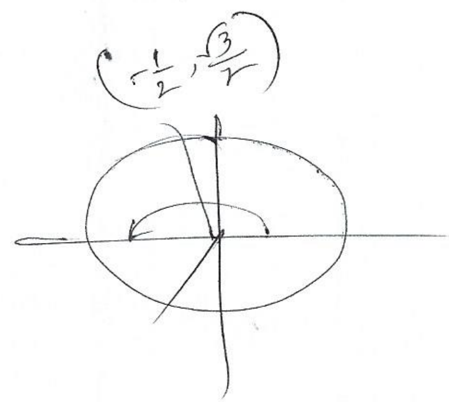
$\sin\theta = 1/2$

QI, QII

$\frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\right\}$

Solve: $0 \le \theta < 2\pi$

$3\cos\theta + 3 = 2\sin^2\theta \quad 0 \le \theta < 2\pi$



$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$3\cos\theta + 2\cos^2\theta + 1 = 0$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -\frac{1}{2}, -1 \qquad (-1, 0)$

$\cos^2\theta + \sin\theta = 2$

$1 - \sin^2\theta + \sin\theta - 2 = 0$

$\Rightarrow \sin^2\theta - \sin\theta + 1 = 0$

$\sqrt{b^2 - 4ac} > 0$ for having solutions.

$\sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}$

$= \sqrt{1-4} = \sqrt{-3}$ no real solutions.

ex. solve:

$$5\sin x + x = 3$$

$$y_1 = 5\sin x + x$$

$$y_2 = 3$$