ex: find inverse function $f^{-1}$
*find range/domain of $f^{-1}$*

\[ y = \cos(x) \]
\[ x = \cos^{-1}(y) \]
\[ -\frac{\pi}{2} \leq \cos^{-1}(y) \leq \frac{\pi}{2} \]

\[ 0 \leq x \leq \frac{\pi}{3} \]
\[ -\frac{\pi}{2} \leq \cos^{-1}(x) \leq \frac{\pi}{2} \]
\[ 0 \leq \frac{\pi}{3} \cos^{-1}(x) \leq \frac{\pi}{2} \]

\[ \text{domain of } f \]
\[ 0 \leq x \leq \frac{\pi}{3} \]
\[ -1 \leq \cos x \leq 1 \]
\[ -1 \leq \cos y \leq 1 \]
\[ 0 \leq \cos 3y \leq 2 \]
\[ \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \]
ex: solve

\[3 \sin^{-1} x = \pi\]
\[\sin^{-1} x = \frac{\pi}{3}\]
\[x = \sin \frac{\pi}{3}\]
\[x = \sin 60^\circ\]
\[x = \frac{\sqrt{3}}{2}\]

ex: find exact value

\[\sin(\tan^{-1} \frac{1}{2})\]

\[= \sin \theta = \frac{\sqrt{5}}{5}\]

\[\theta = \tan^{-1} \frac{1}{2}\]
\[\tan \theta = \frac{1}{2} = \frac{y}{x}\]

\[\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\]
\[\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\]

not unit circle.

\[y + 1 = r\]
\[r = \sqrt{5}\]
\[
\alpha_x: \quad \cos (\sin^{-1} \left( -\frac{1}{3} \right))
\]
\[
\cos \theta
\]
\[
\theta = \sin^{-1} \left( -\frac{1}{3} \right) = \frac{\pi}{2} + \frac{\pi}{6}
\]
\[
\cos \theta = \frac{3}{\sqrt{10}}
\]

Define \(\csc, \sec, \cot\)

\[
y = \sec^{-1} x
\]
\[
y = \sec x
\]
\[
\csc, \sec, \cot
\]

\(\alpha_{y1} > 1\), \(0 \leq y \leq x\), \(\theta = \frac{\pi}{2}\).

\[
\cos \cdot \sin \circ \text{ and } \frac{1}{\cos} = \sec \rightarrow \frac{1}{0} = \text{undefined}.
\]
\[
1 \times 1 \geq 1, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \quad y \neq 0
\]

\[
b = \csc^{-1} x, \quad x = \cot y.
\]

\[
= \arccsc x
\]

\[
-\infty \leq x \leq \infty, \quad 0 < y < \pi
\]

**ex. Find exact value**

\[
\theta = \csc^{-1} 2
\]

\[
\csc \theta = 2
\]

\[
\frac{1}{\sin \theta} = 2
\]

\[
\sin \theta = \frac{1}{2}
\]

\[
\theta = \frac{\pi}{6}
\]

\[
\frac{-\pi}{2} < \theta < \frac{\pi}{2}
\]

\[
\text{Q I, Q IV}
\]

**ex. Rewrite expression with } u, \text{ but no trig } f(x).**

\[
\sin (\tan^{-1} u) = \frac{u}{\sqrt{1 + u^2}}
\]

\[
\theta = \tan^{-1} u
\]

\[
u = \tan \theta.
\]
\[
\sin \theta \cos \theta \over \cos \theta = \tan \theta \cdot \cos \theta \\
= \tan \theta \cdot \frac{1}{\sec \theta} \\
= \frac{\tan \theta}{\sec \theta} \\
= \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}
\]

\[
\cos^2 \theta + \sin^2 \theta = 1 \\
\sin^2 \theta + \cos^2 \theta = 1 \\
\sin \theta = \sqrt{1 - \mu^2}
\]

\[
\tan(\cos^{-1} \mu) \\
= \tan \theta \\
= \frac{\sin \theta}{\cos \theta} \\
= \frac{\sqrt{1 - \mu^2}}{\mu}
\]