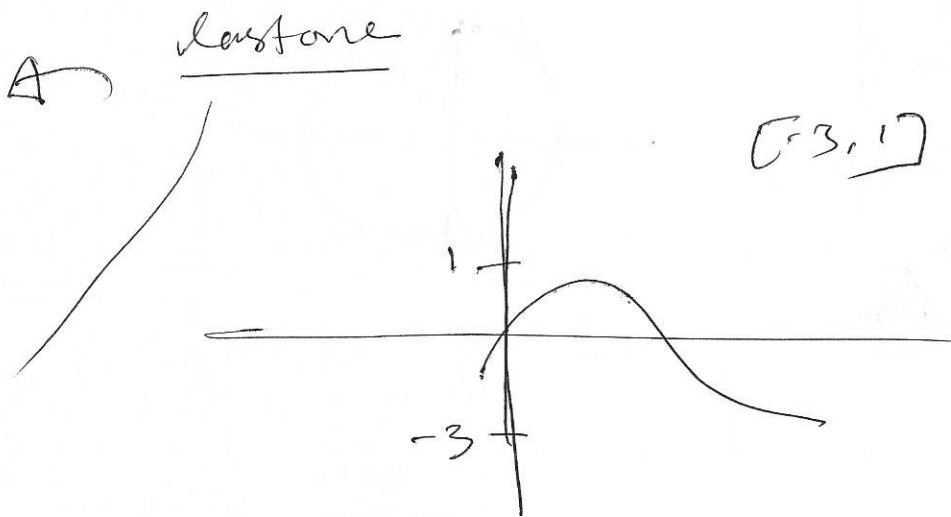


ex: find inverse function  $f^{-1}$   
 find range/domain of  $f^{-1}$

10/29/2018



$$-1 \leq \sin x \leq 1$$

$$-2 \leq 2 \sin x \leq 2$$

$$-3 \leq 2 \sin x - 1 \leq 1$$

ex:  $f(x) = -2 \cos(3x)$

$$y = -2 \cos(3x)$$

$$x = -2 \cos(3y)$$

$$-\frac{x}{2} = \cos(3y)$$

$$3y = \cos^{-1}\left(-\frac{x}{2}\right)$$

$$y = \frac{1}{3} \cos^{-1}\left(-\frac{x}{2}\right)$$

Domain of

$$0 \leq x \leq \frac{\pi}{3}$$

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \cos 3x \leq 1$$

$$-2 \leq -2 \cos 3x \leq 2$$

$$[-2, 2]$$

ex: solve

$$3 \sin^{-1} x = \pi$$

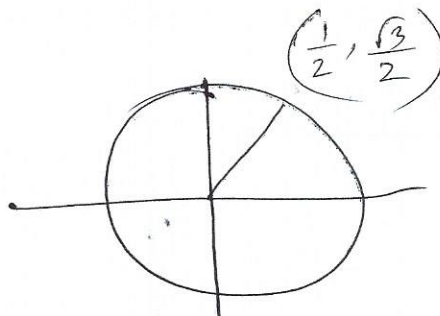
$$\sin^{-1} x = \frac{\pi}{3}$$

$$x = \sin \frac{\pi}{3}$$

$$x = \sin 60^\circ$$

$$x = \frac{\sqrt{3}}{2}$$

9



ex. find exact value

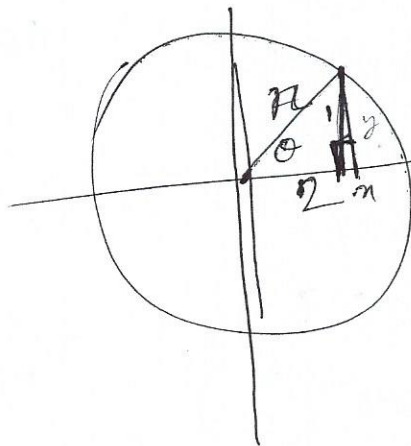
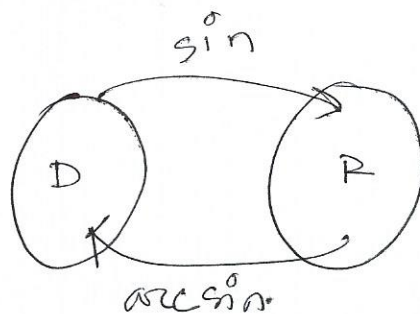
$$\sin(\tan^{-1} \frac{1}{2})$$

$$= \sin \theta = \frac{y}{r}$$

$$\theta = \tan^{-1} \frac{1}{2}$$

$$\tan \theta = \frac{1}{2} = \frac{y}{x}$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



→ not mit übele.

$$4 + 1 = r^2$$

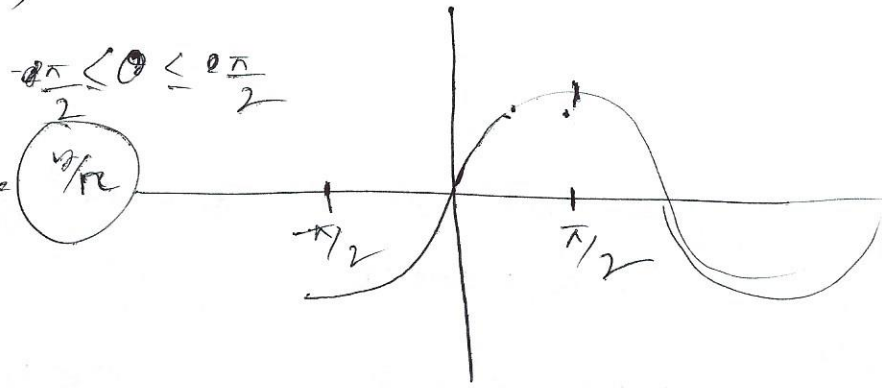
$$r = \sqrt{5}$$

Qx!  $\cos(\sin^{-1}(-\frac{1}{3}))$

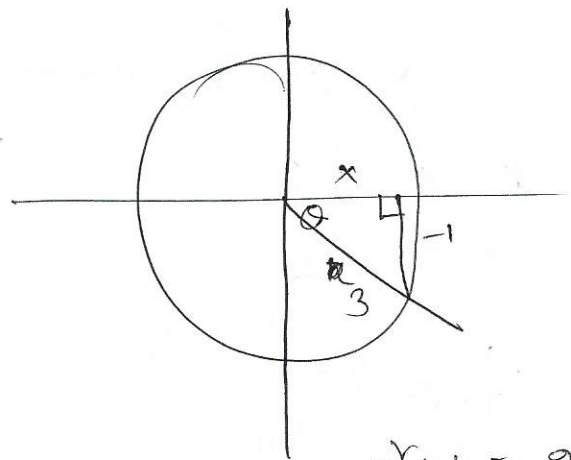
$\cos \theta$

$\theta = \sin^{-1}(-\frac{1}{3}) = \frac{7\pi}{6}$

$\cos \theta = \frac{x}{r}$   
 $= \frac{2\sqrt{2}}{3}$



QI, QIV



$r^2 + 1 = 9$   
 $r^2 = 8$   
 $r = 2\sqrt{2}$

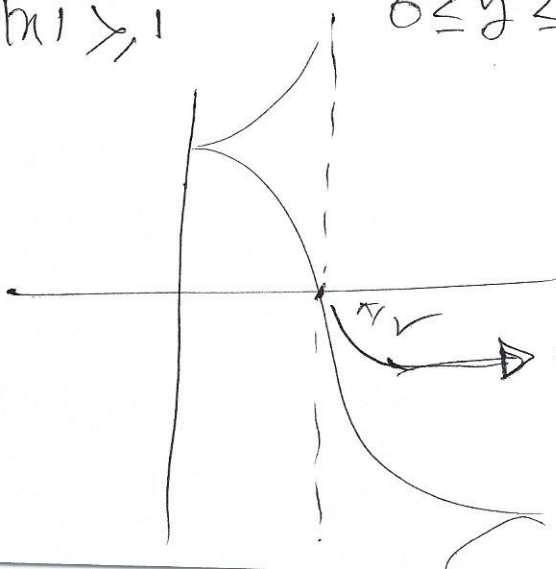
Define  $\csc, \sec, \cot$

$y = \sec^{-1} x$

$x = \sec y$

$\rightarrow = \operatorname{arcsec} x$

$|x| \geq 1$        $0 \leq y \leq \pi$  ,  $y \neq \frac{\pi}{2}$



$\cos \rightarrow 0$  and  $\frac{1}{\cos} = \sec$  no  
 $\frac{1}{0} = \text{undefined}$

$$|x| \geq 1$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y \neq 0$$

$$y = \cot^{-1} x \quad x = \cot y \\ = \operatorname{arccot} x$$

$$-\infty < x < \infty$$

$$0 < y < \pi$$

ex: find exact value

$$\theta = \csc^{-1} 2$$

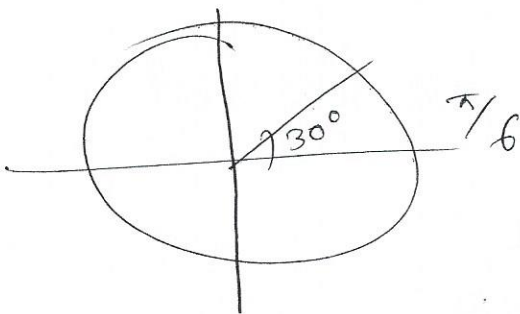
$$\csc \theta = 2$$

$$\frac{1}{\sin \theta} = 2$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ \text{Q I, Q IV}$$



ex Rewrite expression with  $u$ , but no trig  $f(x)$ .  
as (algebraic)

$$\sin(\underbrace{\tan^{-1} u}_{\theta}) = \sin \theta$$

$$\theta = \tan^{-1} u$$

$$u = \tan \theta$$

$$\sin \theta \frac{\cos \theta}{\cos \theta} = \tan \theta \cdot \cos \theta.$$

$$= \tan \theta \cdot \frac{1}{\sec \theta}$$

$$= \frac{\tan \theta}{\sec \theta}$$

$$= \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$$

$$= \frac{u}{\sqrt{1 + u^2}}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sec \theta = \sqrt{1 + \tan^2 \theta}$$

$$= \sqrt{1 + u^2}$$

$$\tan(\underbrace{\cos^{-1} u}_{\theta})$$

$$\tan \theta$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sqrt{1 - u^2}}{u}.$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$u^2 + \sin^2 \theta = 1$$

$$\sin \theta = \sqrt{1 - u^2}$$

