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MATH 1350
Math for Elem. School Teachers
Dr. Alm 11-13-18

Test Teusday on 11-20-18

Decimal Expansion:

$$\frac{4}{7} = \left(\frac{4\overbrace{000000}^{\text{6 zeros} \rightarrow \text{represents the period length}}}{7}\right) \times \frac{1}{\underbrace{1000000}_{\text{6 zeros to cancel}}}$$

the 6 zero's on the numerator; so non theses are equivalent fractions.

$$= \left(571428 + \frac{4}{7}\right) \times \frac{1}{10^6}$$

$$= \frac{571428}{10^6} + \frac{4}{7} \times \frac{1}{10^6}$$

$$= 0.571428 + \frac{4}{7} \times \frac{1}{10^6}$$

$$\begin{array}{r}
571428 \\
7 \overline{)4000000} \\
\underline{-35} \\
50 \\
\underline{49} \\
10 \\
\underline{-7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
4
\end{array}$$

So, $\frac{4}{7} = \underbrace{0.571488}_{\text{decimal approximation}} + \underbrace{\frac{4}{7} \times \frac{1}{10^6}}_{\text{error}}$

This means, $4000000 = 571428 \times 7 + 4$

and also,

$$\frac{4000000}{7} = \left(571428 + \frac{4}{7}\right)$$

$\frac{4}{7} \simeq 0.571488$ (approximately),

w/ error $\frac{4}{7} \times \frac{1}{10^6}$

Question: Approximate $\frac{1}{13}$ by one period of its decimal expansion, and give the error.

$$\begin{array}{r}
\overbrace{0769230}^{\text{seven zeros}} \\
13 \overline{)10000000} \\
\underline{91} \\
90 \\
\underline{-78} \\
120 \\
\underline{117} \\
30 \\
\underline{-26} \\
40 \\
\underline{-39} \\
10
\end{array}$$

← Keep adding zeros until a repeated pattern emerges,

→ repeated remainder

Class Discussion:

Stop at remainder $=1$

$$\frac{1}{13} = \frac{1000000}{13} \times \frac{1}{10^6}$$

$$= \left(76923 + \frac{1}{13}\right) \times \frac{1}{10^6}$$

$$= \frac{76923}{10^6} + \frac{1}{13} \times \frac{1}{10^6}$$

$$= \underbrace{0.076923}_{\text{repeated terms}} + \frac{1}{13} \times \frac{1}{10^6}$$

A terminating ~~fraction~~ decimal expansion of a fraction will not have a remainder, thus no error term.

Ex.

$$\begin{array}{r} 125 \\ 8 \overline{)1000} \\ \underline{-8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

So, $\frac{1}{8} = \frac{1000}{8} \times \frac{1}{10^3} = 125 \times \frac{1}{10^3} = \frac{125}{10^3} = .125$

Special Case: The repeated remainder does not have to be the 1st.

$\frac{5}{12}$

$$\begin{array}{r} 416 \\ 12 \overline{)5000} \\ \underline{-48} \\ 20 \\ \underline{-12} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

So, $\frac{5}{12} = \frac{5000}{12} \times \frac{1}{10^3}$

$$= \left(416 + \frac{8}{12}\right) \times \frac{1}{10^3}$$

$$= \frac{416}{10^3} + \frac{8}{12} \times \frac{1}{10^3}$$

$$= 0.416 + \underbrace{\frac{2}{3} \times \frac{1}{10^3}}$$

$\frac{2}{3} = .\overline{6}$ which accounts for the repeated 6's in this decimal expansion.