*Standard deviation*

Standard deviation of a discrete random variable

\[ \sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)} \]

\[ \mu_X = \sum x_i \cdot P(x_i) \]

\[ \sigma_X = \sqrt{\sum x_i^2 \cdot P(x_i) - \mu_X^2} \]

Variance: \( \sigma_X^2 = 0.5 \)

**Binomial distribution**

Binomial probability distribution formula.

Criteria for a B.P. experiment:

1. The experiment is performed a fixed number of times. Each repetition of the experiment is called a trial.
2. The trials are independent.
3. For each trial, there are two mutually exclusive outcomes, success or failure.
4. The probability of success for each trial is the same.
Notation for a binomial probability experiment:

1. There are $n$ independent trials.
2. Let $p$ denote the probability of success, $1-p$ is the probability of failure.
3. Let $X$ denote the number of successes in $n$ independent trials: $\{X | X = 0, 1, 2, \ldots, n\}$

The probability of obtaining $x$ success in $n$ independent trials is given by

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n.$$
1. Determine which of the following probability experiments qualify as binomial experiments. For those that are binomial experiments, identify the number of trials, probability of success, probability of failure, and possible values of the random variable $X$.

   a. An experiment in which a basketball player who historically makes 80% of his free throws is asked to shoot three free throws, and the number of free throws made is recorded.

   b. According to a recent Harris Poll, 28% of Americans state that chocolate is their favorite flavor of ice cream. Suppose a simple random sample of size 10 is obtained and the number of Americans who choose chocolate as their favorite ice cream flavor is recorded.

   c. A probability experiment in which three cards are drawn from a deck without replacement and the number of aces is recorded.
2. According to CTIA, 41% of all U.S. households are wireless-only households (no landline). In a random sample of 20 households, what is the probability that
   a. Exactly 5 are wireless-only?

   \[ p(x=5) = \binom{20}{5} (0.41)^5 (0.59)^20-5 \]

   \[ = 6.5\% \]

   b. Fewer than 3 are wireless only?

   \[ P(0) + P(1) + P(2) \]

   \[ = 0.000026 + 0.00363 + 0.06339 = 0.06728 \]

   c. At least 3 are wireless only?

   \[ 1 - P(x<3) \]

   \[ 1 - 0.06728 = 99.72\% \]

   d. The number of households that are wireless-only is between 5 and 7, inclusive?

   \[ P(5) + P(6) + P(7) \]
3. According to CTIA, 41% of all U.S. households are wireless-only households. In a simple random sample of 300 households, determine the mean and standard deviation number of wireless-only households.

\[
\begin{align*}
\text{Mean} & \quad \mu_X = np = 300 \times 0.41 = 123.0 \\
\text{Standard Deviation} & \quad \sigma_X = \sqrt{np(1-p)} = \sqrt{123 \times 0.59} \approx 8.5
\end{align*}
\]