Probability

Tuesday, November 27, 2018 12:47 PM

Dec. 11th 11:00-1:30 – Final exam (Cumulative)

Probability: The probability of a given event, E, is

$$P(E) = \frac{\#\text{ of ways } E \text{ occurs}}{\text{total number of possibilities}}$$

example: probability of rolling a 6 on a 6-sided die.

$$P(\text{roll a 6}) = \frac{1 \text{ way to get a 6}}{6 \text{ possible rolls}} = \frac{1}{6}$$

example: probability of rolling a sum of 8 when rolling 2 six-sided dice where one is blue and one is red.

Ways to roll sum of 8

Blue	Red
2	6
3	5
4	4
5	3
6	2

5 ways to roll 8 & 6·6 different ways to roll a sum

$$P(\text{sum of 8}) = \frac{5}{6 \cdot 6} = \frac{5}{36}$$

Facts: For any event E,

$$0 \leq P(E) \leq 1$$

impossible events

the event always happens

ex. impossible event

I have a bag of 6 blue marbles

What is the probability of pulling a red marble out of the bag?

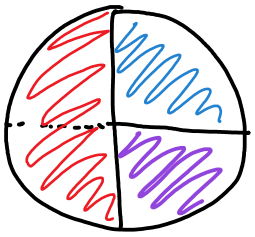
$P(\text{red}) = \frac{0}{6} = 0$

ex. always happens

What is the probability of pulling a blue marble out of the bag?

$P(\text{blue}) = \frac{6}{6} = 1$

ex.



$P(\text{blue}) = \frac{1}{4}$

$P(\text{purple}) = \frac{1}{4}$

$P(\text{red}) = \frac{2}{4} = \frac{1}{2}$

these events are not equally likely.

The probabilities are not equal.

$P(\text{blue or purple}) = \frac{1+1}{4} = \frac{2}{4}$

"or" uses +

as long as these events don't overlap.

$P(\text{blue and purple}) = 0$ ← we can't spin the wheel and get both blue and purple → impossible event

both have to happen at the same time.

Probability that an event does not happen

E = event

$P(\text{not } E) = 1 - P(E)$

$P(\text{not red}) = 1 - \frac{1}{2} = \frac{2}{2}$

ex. Unfair die which is 2x as likely to land on 1 as any other number

$P(1) = ?$

$P(2) = ?$

We know $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

these all have the same chance to roll.

we also know $P(1) = 2 \cdot P(2)$

mathy explanation

$P(1) + 5 \cdot P(2) = 1$

$2 \cdot P(2) + 5 \cdot P(2) = 1$

$7P(2) = 1$

$P(2) = \frac{1}{7}$

$P(1) = 2 \cdot P(2) = 2 \cdot (\frac{1}{7}) = \frac{2}{7}$

Another way to look at this problem:

Possible rolls: 1, 2, 3, 4, 5, 6

you're twice as likely to roll this, so add another 1

1, 1, 2, 3, 4, 5, 6

Now $P(1) = \frac{2}{7}$, $P(2) = \frac{1}{7}$

What if 1 is 3 times as likely to appear as any other number?

1, 1, 1, 2, 3, 4, 5, 6

$P(1) = \frac{3}{8}$ $P(2) = \frac{1}{8}$

each # is equally likely to roll

ex. Roll two fair six-sided dice

$P(\text{sum of } 12) = \frac{1}{36}$ (6,6)

$P(\text{sum of } 10) = \frac{3}{36}$ (5,5; 4,6; 6,4)

$P(\text{sum of } 11) = \frac{2}{36}$ (5,6; 6,5)

$P(\text{Sum is odd}) = P(\text{sum is } 1 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 9 \text{ or } 11)$

$= \frac{18}{36}$

0 ways

1 way: 1,4; 4,1; 2,3; 3,2

2 ways: 1,2; 2,1

3 ways: 2,1; 1,2; 3,6; 6,3; 4,5; 5,4

3 ways: 1,6; 6,1; 2,5; 5,2; 3,4; 4,3

1 way: 5,6; 6,5

Count them all

ex. Roll 3 fair 6-sided dice (green, blue & red dice)

P(Sum of 3)