Probability

Dec. 11th 11:00-1:30 - Final exam (Cumulative)

Probability: The probability of a given event, E, is

\[ P(E) = \frac{\text{# of ways } E \text{ occurs}}{\text{total number of possibilities}} \]

Example: Probability of rolling a 6 on a 6-sided die.

\[ P(\text{roll a 6}) = \frac{1 \text{ way to get a 6}}{6 \text{ possible rolls}} = \frac{1}{6} \]

Example: Probability of rolling a sum of 8 when rolling 2 six-sided dice where one is blue and one is red.

Ways to roll sum of 8:

\begin{align*}
2, 6 \\
3, 5 \\
4, 4 \\
5, 3 \\
6, 2 \\
\end{align*}

& 6 \times 6 \text{ different ways to roll a sum}

\[ P(\text{sum of 8}) = \frac{5}{36} \]

Facts: For any event E, 
\[ 0 \leq P(E) \leq 1 \]
**Ex. impossible event**

I have a bag of 6 blue marbles.

What is the probability of pulling a red marble out of the bag?

\[ P(\text{red}) = \frac{0}{6} = 0 \]

**Ex. always happens**

What is the probability of pulling a blue marble out of the bag?

\[ P(\text{blue}) = \frac{6}{6} = 1 \]

**Diagram:**

\[ P(\text{blue}) = \frac{1}{4} \]
\[ P(\text{purple}) = \frac{1}{4} \]
\[ P(\text{red}) = \frac{2}{4} = \frac{1}{2} \]

These events are not equally likely. The probabilities are not equal.

\[ P(\text{blue or purple}) = \frac{\frac{1}{4} + \frac{1}{4}}{4} = \frac{2}{4} \]

"or" uses + as long as these events don't overlap.

\[ P(\text{blue and purple}) = 0 \]

we can't spin the wheel and get both blue and purple \(\rightarrow\) impossible event.

both have to happen at the same time.

Probability that an event does not happen.
Unfair die which is 2x as likely to land on 1 as any other number.

\[ P(1) = \frac{2}{7}, \quad P(2) = \frac{1}{7} \]

Another way to look at this problem:
Possible rolls: 1, 2, 3, 4, 5, 6
You're twice as likely to roll this, so add another 1
1, 1, 2, 3, 4, 5, 6

Now \[ P(1) = \frac{2}{7}, \quad P(2) = \frac{1}{7} \]
1. Roll two fair six-sided dice

\[ P(\text{sum of 12}) = \frac{1}{36} \quad P(\text{sum of 10}) = \frac{3}{36} \]

\[ P(\text{sum of 11}) = \frac{2}{36} \]

\[ P(\text{sum is odd}) = P(\text{sum is 1 or 3 or 5 or 7 or 9 or 11}) = \frac{18}{36} = \frac{1}{2} \]

2. Roll 3 fair six-sided dice (green, blue & red dice)

\[ P(\text{sum of 3}) \]