Linear Inequalities in one Variables (or first degree)

- less than
- less than or equal to
- greater than
- greater than or equal to

\[ Ax + B < C \]

where \( A, B, C \) are real numbers with \( A \neq 0 \)

\[ x < 5 \]

interval notation \((-\infty, 5)\)

Addition Property of Inequalities
If \( A, B \) and \( C \) are real numbers, then the inequalities \( A < B \) and \( A + C < B + C \) are equivalent.

\[ x - \frac{3}{3} < -9 \\
   +3 +3 \\
   \frac{x}{-6} \]

check \(-7 - 3 < -9\)
\(-10 < -9\) time

\((-\infty, -6)\)
Multiplication property of Inequality

If \( A, B \) and \( C \) are real numbers with \( C \neq 0 \), then the inequalities \( A < B \) and \( AC < BC \) are equivalent if \( c > 0 \) and \( A < B \) and \( AC > BC \) are equivalent if \( c < 0 \).

\[
3 < 4
\]
\[
(-1)(3) \quad 4(-1)
\]
\[
-3 > -4
\]

\[
\frac{5x}{5} \leq \frac{-10}{5}
\]
\[
x \leq -2
\]

Interval notation : \((-\infty, -2]\)

\[
\frac{-9x}{-9} \leq \frac{81}{-9}
\]
\[
x \geq 9
\]

Interval notation : \((9, \infty)\)
Five times \( x \) is greater than or equal to \(-23\).
\[ 5x \geq -23 \]

To solve:
\[
\frac{5x}{5} \geq \frac{-23}{5}
\]
\[ x \geq -\frac{23}{5} \]

The difference of twice a number and 6 is greater than \(-27\).
\[ 2n - 6 > -27 \]

To solve:
\[
\frac{2n}{2} > \frac{-27}{2} + 6
\]
\[ n > -\frac{21}{2} \]
\[ n > -10.5 \]

\[ (-10.5, \infty) \]

Compound inequality (or 3-part inequality)
\[ 3 < x \leq 8 \]
\[ x > 3 \]

Read as:
\[ x > 3 \text{ and } x \leq 8 \]

\[ x > 3 \text{ and } x \leq 8 \]

Simple way to write \((3,8]\)
For pre-cal

\((x+1)(x-3)(x-8) \geq 0\)

Boundary points

\[ x = -1 \quad x = 3 \quad x = 8 \]

<table>
<thead>
<tr>
<th>True</th>
<th>True</th>
<th>False</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3</td>
<td>8</td>
<td></td>
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</tbody>
</table>

Test \( x = -2 \)

\[ (+) (-) (-) \geq 0 \]

\[ + \geq 0 \] \text{True}

Test \( x = 0 \)

\[ (+) (-) (-) \geq 0 \]

\[ + \geq 0 \] \text{True}

Test \( x = 4 \)

\[ (+) (+) (-) \geq 0 \]

\[ - \geq 0 \] \text{False}

Test \( x = 9 \)

\[ (+) (+) (+) \geq 0 \]

\[ + \geq 0 \] \text{True}

Solution: \((-\infty, 3] \cup [8, \infty)\)
Graph

\[ x \leq 6 \text{ and } x \geq -1 \]

\[ (-1, 6) \]

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Solve the compound inequality:

\[ 3x + 4 \geq -14 \] and \[ 2x + 3 \leq 7 \]

\[ \begin{align*}
3x & \geq -18 \\
\frac{3x}{3} & \geq \frac{-18}{3} \\
x & \geq -6
\end{align*} \]

\[ \begin{align*}
2x + 3 & \leq 7 \\
2x & \leq 4 \\
\frac{2x}{2} & \leq \frac{4}{2} \\
x & \leq 2
\end{align*} \]

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\[ [-6, 2] \]