

1.8. Operations on Functions.

Addition: $f(x) = x^2 - 4x + 3$.

$$g(x) = 2x + \sqrt{x} - 5.$$

$$(f+g)(x) \text{ or } f(x) + g(x) \text{ or } f+g.$$

$$(f+g)(x) = (x^2 - 4x + 3) + (2x + \sqrt{x} - 5).$$

$$(f+g)(x) = x^2 - 2x - 2 + \sqrt{x}.$$

Domain of $(f+g)(x)$

Let A represent the domain of f and B represent the domain of g .

$$\text{Domain of } (f+g)(x) = A \cap B.$$

$$A: (-\infty, \infty)$$

$$B: [0, \infty)$$

$$A \cap B = \text{Domain of } (f+g)(x) = [0, \infty).$$

Subtraction:

$$(f-g)(x) = f(x) - g(x) = f-g.$$

$$f(x) = 2x - 3, \quad g(x) = x^2 + 4x - 7.$$

$$g-f = (x^2 + 4x - 7) - (2x - 3).$$

$$= x^2 + 4x - 7 - 2x + 3.$$

$$= x^2 + 2x - 4$$

Domain:

Domain of $g \Rightarrow D: (-\infty, \infty)$.

" " $f \Rightarrow D: (-\infty, \infty)$.

$D_{g \cdot f} = (-\infty, \infty)$.

Multiplication:

$$(fg)(x) = fg = (f \cdot g)(x) = f(x)g(x).$$

$$f(x) = 2x^2 - 5x + 1, \quad g(x) = 2x + 3.$$

$$(f \cdot g)(x) = (2x^2 - 5x + 1)(2x + 3).$$

$$= 4x^3 + 6x^2 - 10x^2 - 15x + 2x + 3.$$

$$= \boxed{4x^3 - 4x^2 - 13x + 3}$$

$$D: (-\infty, \infty).$$

$$f(x) = \sqrt{x-3}$$

$$D: [3, \infty)$$

$$g(x) = \frac{1}{x-7}$$

$$D: (-\infty, 7) \cup (7, \infty).$$

$$(g \cdot f)(x).$$

$$D: [3, 7) \cup (7, \infty)$$

$$(g \cdot f)(x) = \frac{1}{x-7} \cdot \frac{\sqrt{x-3}}{1} = \boxed{\frac{\sqrt{x-3}}{x-7}}$$

Division:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{f}{g}.$$

Example: $f(x) = x^2 - 3x + 5$, $g(x) = x - 1$.

$$D: (-\infty, \infty).$$

$$D: (-\infty, \infty).$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 3x + 5}{x - 1}.$$

$$D: (-\infty, 1) \cup (1, \infty).$$

$$f(x) = \sqrt{x+3} - 1 \quad g(x) = x+2.$$

$$(f \cdot g)(6)?$$

$$(f \cdot g)(6) = f(6) \cdot g(6).$$

$$= (\sqrt{6+3} - 1)(6+2).$$

$$= (2)(8) = \boxed{16}.$$