Division Algorithm

Suppose that \( f(x) \) and \( d(x) \) be two polynomials where \( d(x) \neq 0 \) and the degree of \( d(x) \) is less than or equal to the degree of \( f(x) \). Then there exists unique polynomials \( q(x) \) and \( r(x) \) such that

\[
\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}, \text{ and } f(x) = q(x).d(x) + r(x)
\]

Where either the degree of \( r(x) \) is less than \( d(x) \), or \( r(x) = 0 \).

Note \( f(x) \) is the dividend, \( d(x) \) is the divisor, \( q(x) \) is the quotient, and \( r(x) \) is the remainder.

Remainder Theorem

If the polynomial \( f(x) \) is divided by \( (x - c) \), then the remainder is \( f(c) \).

Factor Theorem

Let \( f(x) \) be a polynomial.

1. If \( f(c) = 0 \), then \( (x - c) \) is a factor of \( f(x) \).
2. If \( (x - c) \) is a factor of \( f(x) \), \( f(c) = 0 \).
9) \( (3x^3 - 12x^2 - 10) \div (x-1) \)

\[ q(x) \rightarrow \left[ 3x^2 + x + 4 \right] + \frac{6}{x-1} \]

\[ x-1 \]
\[ \left[ 3x^3 - 12x^2 + 0x - 10 \right] \]
\[ - (3x^3 - 12x^2) \]
\[ = \]
\[ x^2 \]
\[ + 0x - 10 \]
\[ - (x^2 - 4x) \]
\[ = \]
\[ 4x - 10 \]
\[ - (4x - 16) \]
\[ = \]
\[ \frac{6}{x-1} = \frac{f(x)}{d(x)} \]

\[ q(x) = 3x^2 + x + 4 + \frac{6}{x-1} \]

15) \( (x^5 + 4x^4 + 18x^2 - 20x - 10) \div (x^2 + 5) \)

\[ x^3 + 4x^2 - 5x - 2 + \frac{5x}{x^2 + 5} \]

\[ x^2 + 5 \]
\[ \left[ x^5 + 4x^4 + 0x^3 + 18x^2 - 20x - 10 \right] \]
\[ - (x^5 + 0x^4 + 5x^3) \]
\[ = 4x^4 - 5x^3 + 18x^2 - 20x - 10 \]
\[ - (4x^4 + 0x^3 + 20x^2) \]
\[ = -5x^3 - 2x^2 - 20x - 10 \]
\[ - (-5x^3 + 0x^2 - 25x) \]
\[ = -2x^2 + 5x - 10 \]
\[ - (-2x^2 + 0x - 10) \]
\[ = \]
\[ \left[ x^5 + x - \frac{5}{x^2 + 5} \right] \]

When the degree of the remaining terms is strictly less than the largest degree in the divisor, you stop and call the rest your reminder, \( r(x) \).
19) \[
\frac{x^3 - 2.7}{x - 3} = \frac{x^2 + 3x + 9}{x^3 + 0x^2 + 0x - 2.7}
\]

Factoring theorem: if \( f(x) = 0 \), then \( f(x) = q(x)d(x) \), so \( x^3 - 2.7 = (x^2 + 3x + 9)(x - 3) \)

19) using Synthetic division

\[
\begin{array}{c|ccc}
1 & 0 & 0 & -2.7 \\
3 & 3 & 3 & 9 \\
\hline
1 & 3 & 9 & 0 \\
\end{array}
\]

* S.D. can only be done when divisor is linear, or its degree is 1. \((x^1)\)
31) \((4 - 8x - 3x^2 - 5x^4) \div (x - 2)\)

\[
\begin{array}{cccc}
& -5x^4 & +0x^3 & -3x^2 & -8x & +4 \\
\underline{2} & & -10 & -20 & -46 & -108 \\
& -5 & -10 & -23 & -54 & -104 \\
\end{array}
\]

\[-5x^3 - 10x^2 - 23x - 54 - \frac{104}{x-2}\]

we're doing this to see if problems are factorable, which they are if you get no remainder.

34) \((2x^5 + 13x^4 - 3x^3 - 58x^2 - 20x + 24) \div (x - 2)\)

\[
\begin{array}{cccc}
& 2 & 13 & -3 & -58 & -20 & 24 \\
\underline{2} & & 17 & 31 & 4 & -12 & \underline{0} \\
2 & 4 & 17 & 31 & 4 & -12 & \underline{r(x) = 0}
\end{array}
\]

This means \(f(x)\) is factorable.
43) \[ f(x) = 2x^4 + x^3 - 49x^2 + 79x + 15 \]

a) Find \( f(-1) \).

\[
f(-1) = 2(-1)^4 + (-1)^3 - 49(-1)^2 + 79(-1) + 15
= 2 - 1 - 49 - 79 + 15 = -112
\]

Same as if we did S. D. for \( x = 1 \) is 0.

\[ x = -1 \]

\[
\begin{array}{c|cccc}
-1 & 2 & 1 & -49 & 79 & 15 \\
\hline
1 & 2 & 1 & -48 & 127 & 112 \\
\end{array}
\]

b) Find \( f(3) \). Can either plug in 3 in the equation, or do S. D. for \( x = 3 \). If you have a calculator, first step is easily replicable to what you did for a).

\[
f(3) = 0
\]

\[
\begin{array}{c|cccc}
3 & 2 & 1 & -49 & 79 & 15 \\
\hline
1 & 6 & 21 & -84 & -15 \\
\end{array}
\]

\[ f(x) = (x - 3)(2x^3 + 7x^2 - 28x - 5) \]