

II. Multiplying a single-digit # by a multi-digit #

$$\begin{array}{r} \text{Ex.} \quad 2 \\ 2187 \\ \times 3 \\ \hline 561 \end{array}$$

This works b/c $187 \cdot 3 = (\cancel{100} + 80 + 7) \cdot 3$

$$\begin{aligned} &= (100 \cdot 3) + (80 \cdot 3) + (7 \cdot 3) && \text{used distributive law} \\ &= 300 + 240 + 21 && \text{\& now these are} \\ &= 300 + (200 + 40) + (20 + 1) && \text{essentially single-} \\ & && \text{digit mult.} \end{aligned}$$

\downarrow \downarrow
 the other corresponds
 2 that was to the first 2
 carried...! that was carried!
 100's place 10's place

$$\begin{aligned} &= (300 + 200) + (40 + 20) + 1 \\ &= 500 + 60 + 1 \\ &= \underline{561} \end{aligned}$$

Example using Aliens w/ 4 fingers!

So, symbols are 0, 1, 2, 3

Multiplication Table:

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0		10	12
3	0			21

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33

$$\begin{array}{r} 2 \\ 232 \\ \times 3 \\ \hline \underline{2022} \end{array}$$

Expanded form:

$$\begin{aligned} (200 + 30 + 2) \cdot 3 &= (200 \cdot 3) + (30 \cdot 3) + (2 \cdot 3) \\ &= 1200 + 210 + \cancel{21} = (1000 + 200) + (200 + 10) + (\cancel{20 + 2}) \\ &= 1000 + (200 + 200) + (20 + 10) + 2 = 1000 + 1000 + 20 + 2 \\ &= \underline{2022} \end{aligned}$$

Another Example:

$$\begin{array}{r} 11 \\ 333 \\ \times 2 \\ \hline 1332 \end{array}$$

$$\begin{array}{cccc} 00 & 01 & 02 & 03 \\ 10 & 11 & 12 & 13 \\ 20 & 21 & 22 & 23 \\ 30 & 31 & 32 & 33 \end{array}$$

$$\begin{aligned} \text{Expanded: } (300 + 30 + 3) \cdot 2 &= (300 \cdot 2) + (30 \cdot 2) + (3 \cdot 2) \\ &= 1200 + 120 + 12 \\ &= (1000 + 200) + (100 + 20) + (10 + 2) \\ &= 1000 + 300 + 30 + 2 \\ &= 1332 \end{aligned}$$

III. Multi-digit Multiplication

$$\begin{array}{r} ~~137~~ \\ ~~137~~ \\ ~~137~~ \\ ~~137~~ \\ ~~137~~ \\ \times 12 \\ \hline 174 \\ + 37 \\ \hline 444 \end{array}$$

Expanded:

$$\begin{aligned} 37 \cdot 12 &= (30 + 7) \cdot (10 + 2) \\ &= [(30 + 7) \cdot 10] + [(30 + 7) \cdot 2] \\ &\quad \uparrow \\ &\quad \text{essentially multiplying} \\ &\quad \text{by a single-digit} \\ &= (30 \cdot 10) + (7 \cdot 10) + (30 \cdot 2) + (7 \cdot 2) \\ &= 300 + 70 + 60 + 14 \\ &= 370 + (60 + 10) + 4 \end{aligned}$$

Ex. Similar to # 2 in HW

$$\begin{array}{r} 123,456,789 \\ \times \quad 53 \\ \hline \end{array}$$

$$789 \times 53 = 41817$$

$$456 \times 53 = 24168$$

$$123 \times 53 = 6519$$

$$\begin{array}{r} 41817 \\ 24168000 \\ + 6519000000 \\ \hline \end{array}$$