If \( \frac{m}{n} \) is a fraction, it is said to be in lowest terms if there is no whole number \( c > 1 \) that divides both \( m + n \).

Example: \( \frac{4}{6} \)

\[
\begin{align*}
0 & \quad \frac{1}{6} \quad \frac{2}{6} \quad \frac{3}{6} \quad \frac{4}{6} \quad \frac{5}{6} \quad \frac{6}{6} \\
\hline
0 & \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{3}{3}
\end{align*}
\]

Since \( \frac{4}{6} \) and \( \frac{2}{3} \) are the same point on the number line,

\[
\frac{4}{6} = \frac{2}{3}
\]

How does "cancellation" fit in this?

\[
\frac{4}{6} = \frac{2 \times 2}{3 \times 3} = \frac{2}{3}
\]

Theorem of Equivalent Fraction:

If \( \frac{m}{n} \) is a fraction and \( c \geq 0 \) is a whole number,

\[
\frac{m}{n} = \frac{cm}{cn}
\]

Reason:

For concreteness, let \( c = 5 \).

Then, to get from 0 to \( \frac{m}{n} \), we take \( m \) steps to the right of 0 of length \( \frac{1}{n} \).

If instead we take steps of length \( \frac{1}{5n} \), it will now take \( 5m \) steps to reach \( \frac{m}{n} \).

So,

\[
\frac{m}{n} = \frac{5m}{5n}
\]
Explain to a 5th grader, why $\frac{1}{2} = \frac{5}{10}$.

Write $\frac{3}{5}$ as a decimal fraction and its decimal notation:

$$\frac{3}{5} = \frac{2 \times 3}{2 \times 5} = \frac{6}{10} = 0.6$$

Write $\frac{28}{35}$ as a decimal:

$$\frac{28}{35} = \frac{7 \times 4}{7 \times 5} = \frac{4}{5} = \frac{2 \times 4}{2 \times 5} = \frac{8}{10} = 0.8$$

A Note:

Decimal terminates when,
the fraction is of form.

$$\frac{n}{2^k \cdot 5^j} \quad \text{where} \quad K \text{ and } j \text{ are whole numbers}$$

$$\frac{3}{25} = \frac{3}{5 \times 5} = \frac{2 \times 2 \times 3}{2 \times 2 \times 5 \times 5} = \frac{12}{100}$$
Extra: Euclidean Alg.

\[ \frac{48}{62} \]

\[ 62 = 1 \times 48 + 14 \]

\[ \frac{48}{62} = \frac{3 \times 24}{2 \times 31} = \frac{24}{31} \]

\[ 48 = 3 \times 14 + 2 \]

\[ 14 = 7 \times 2 + 0 \]

Fundamental Fact of Fraction Pairs

Any two fractions can be put "on the same scale", i.e., given a common denominator.

The common denominator can ALWAYS be taken to be the product of the two denominators.

If \( \frac{m}{n} \) and \( \frac{a}{b} \) are the fractions, then

\[ \frac{m}{n} = \frac{mb}{nb} + \frac{a}{b} = \frac{an}{bn} \]

have the same denominator.

Which is bigger?

\[ \frac{4}{7} \text{ or } \frac{3}{5} ? \]

\[ \frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35} \]

\[ \frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35} \]

So, \( \frac{3}{5} > \frac{4}{7} \).