Section 6.2

- Intro Probability notes
- Probability Laws
- Learn what Complement and Mutually Exclusive mean
A **random experiment** is an experiment, trial, or observation that can be repeated numerous times under the same conditions.

The set of all distinct outcomes of the experiment is called the **sample space**, denoted $S$. An **event** is a set of outcomes (what are we looking for in the experiment).

**Examples:** Write out the sample space for the given experiment.

1. A coin is tossed twice in a row.
2. A coin is tossed 3 times in a row.
3. You roll a die once.

**Three types of probability**

1. *The Relative Frequency approach:* Observed Probability. Works best for experiments that can be easily repeated, such as coin toss, rolling a die, and card experiments.

   **Example:** At a local university, you poll a group of 115 students and find that 60 of them are from out-of-state.

   $$\text{Relative Frequency of } A = \frac{\text{number of observed events}}{\text{total number of experiments}}$$

2. *Classical Probability:* Measured as a simple proportion, the number of outcomes of the event divided by the number of outcomes in the sample space. Not always observed.

   $$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes in sample space}}$$

3. *Subjective Probability:* Regards the probability of an event as a measure of the degree of belief that the event has occurred or will occur. Based on experience and life events.

   **Example:** You guess that there is a 65% chance that you will be assigned homework in your Spanish class on Thursday.
Classical Probability Examples

1. You roll a fair, six-sided die. What is the probability of getting a 5?

2. What is the probability of rolling a sum of 10 on a standard pair of six-sided dice?

3. A coin is tossed 3 times. What is the probability of getting three heads?

4. You are going to play mini golf. A ball machine that contains 23 green golf balls, 15 red golf balls, 17 blue golf balls, and 24 yellow golf balls, randomly gives you your ball. What is the probability that you end up with a blue golf ball? Express your answer as a simplified fraction or a decimal rounded to four decimal places.
Probability Laws:

1. A probability of 0 means the event cannot happen.
2. A probability of one means the event must happen.
3. All probabilities must be between 0 and 1 inclusively. The closer the probability is to 0, the less likely the event.
4. The sum of the probabilities of all outcomes must equal 1.

\[ P(\text{Heads}) + P(\text{Tails}) = 1 \]

The complement of an event A is the set of all outcomes in the sample space that are not in A.

**Example:** Describe the complement of the set: 71% of subscribers to a fitness magazine are over the age of 21. 71% over 21, so the 29% that is the complement.

**Probability Law:** The probability of \( A^c \) is

\[ P(A^c) = 1 - P(A) \quad \text{or} \quad P(A^c) = 1 - P(A) \]

**Example:** A newspaper company classifies its customers by gender and location of residence. The research department has gathered data from a random sample of 2113 customers. The data is summarized in the table to the right.

<table>
<thead>
<tr>
<th>Gender and Residence of Customers</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apartment</td>
<td>165</td>
<td>126</td>
</tr>
<tr>
<td>Dorm</td>
<td>218</td>
<td>256</td>
</tr>
<tr>
<td>With Parent(s)</td>
<td>278</td>
<td>272</td>
</tr>
<tr>
<td>Sorority/Fraternity House</td>
<td>95</td>
<td>267</td>
</tr>
<tr>
<td>Other</td>
<td>258</td>
<td>178</td>
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</tbody>
</table>

What is the probability that a customer is not male or does not live in an apartment? Express your answer as a fraction or a decimal number rounded to four decimal places.
Two events are mutually exclusive if they have no outcomes in common.

![Figure 6.2.5](image)

They cannot both happen at the same time. Ex: a door being open and closed are mutually exclusive.

**Examples:** Are the following events mutually exclusive?

1. Choosing a student who is a history major or a finance major from a nearby university to participate in a research study. (Assume that each student only has one major.)
   - Mutually exclusive b/c each person has one major
2. Choosing a red card or a club out of a standard deck of cards.
   - Mutually exclusive b/c if a card is a club, it is black not red.
3. Choosing a ten or a spade out of a standard deck of cards.
   - Not mutually exclusive b/c there is a 10 of spades so a card can be both.

**Probability Law:** If two events, A and B, are mutually exclusive, then

\[ P(A \cup B) = P(A) + P(B) \]

\[ \text{A or B} \]

**Examples:**

1. A newspaper company classifies its customers by gender and location of residence. The research department has gathered data from a random sample of 2113 customers. The data is summarized in the table to the right.

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What is the probability that a customer is male and lives in an apartment or is female and lives in a dorm? Express your answer as a fraction or a decimal number rounded to four decimal places.

2. A standard pair of six-sided dice is rolled. What is the probability of rolling a sum greater than or equal to 10?
Ex: Newspaper Example from page 4

What is the probability that a customer is not male or does not live in an apartment?

- This is a bit complicated so let's find the complement to see if that's easier.

- A: Not male or Not apt.
  A^c: male and apt

\[ P(A^c) = \frac{165}{2113} \]

So \[ P(A) = 1 - P(A^c) \]

\[ = 1 - \frac{165}{2113} \]

\[ = \frac{2113}{2113} - \frac{165}{2113} \]

\[ = \frac{1948}{2113} \]

Hawks will accept either

\[ \approx 0.9219 \]
\[
E_X:
\begin{align*}
A: & \text{ Choosing a red card } \quad \frac{2}{3} \quad \text{Mutually Exclusive} \\
B: & \text{ Choosing a club } \\
\Pr(A \cup B) &= \Pr(A \text{ or } B) = \Pr(A) + \Pr(B) \\
&= \frac{26}{52} + \frac{13}{52} \\
&= \frac{39}{52} \\
&= 0.75
\end{align*}
\]

\[
\begin{align*}
E_X:
A: & \text{ Male and apt } \quad \frac{3}{3} \quad \text{M.E.} \\
B: & \text{ Female and dorm } \\
\Pr(A \text{ or } B) &= \Pr(A) + \Pr(B) \\
&= \frac{165}{2113} + \frac{256}{2113} \\
&= \frac{421}{2113} \\
& \approx 0.1992
\end{align*}
\]
\[ \text{Ex.} \quad P(\text{sum } \geq 10) = P(10) + P(11) + P(12) \]

\[ \begin{align*}
\text{10} & \quad \text{11} & \quad \text{12} \\
(4,6) & \quad (5,6) & \quad (6,6) \\
(6,4) & \quad (6,5) & \quad (6,6) \\
(5,5) & & \\
\end{align*} \]

\[ = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6} \]

Sample size: 36