# POLYNOMIALS



3/3/20 Tuesday

(1) Distinguished by the # of terms

monomial e.g. $3x$

binomial e.g. $5x+7$

trinomial e.g. $x^2+3x-14$

(2) Distinguished by the degree.

Definition of degree

- The largest exponent in any term of the polynomial in $x$.
  The degree determines the number of ~~s~~solutions when setting the polynomial in $x$ to equal zero and solve.
- If the polynomial involves more than one variables, the degree of the polynomial is defined to be the largest sum of ~~the~~ exponents in any term.

e.g. $7x^5-5x^3+2x-9$, degree is 5

$4a^3b^4c+11ab^3c^2-2abc^3$, degree is $3+4+1=8$

(3) Other terminology:

Definition of standard form.

The polynomial is in descending order of exponents

e.g. $7x^5-5x^3+2x-9$ ← refer to this exam

Definition of leading term:

The first term of the polynomial in standard form.

aka, the term with the highest degree of exponent. — $7x^5$

Definition of leading coefficient:

The coefficient on the leading term. — 7

$7x^{5}$ ← "5" is the exponent.
↳ "x" is the base
7 is coefficient



(4) Requirement for a Polynomial

The exponent of each term must be WHOLE NUMBER
i.e. $\{0,1,2,3,4,5,\cdots\}$

Notice any number raised to the zero power equals 1
e.g. 8 is a polynomial of zero degree

$$8 = 8\cdot 1 = 8\cdot x^{0}$$

$3x^{2}+4x-7+3x^{-1}$ NOT a polynomial because of $-1$

$7x^{\frac{3}{4}}+5x^{\frac{1}{4}}$ NOT a polynomial because of $\frac{3}{4},\frac{1}{4}$

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## FACTORING

① $x^{4}-9 = (x^{2}+3)(x^{2}-3)$

② $x^{6}-25 = (x^{3}+5)(x^{3}-5)$

③ $a^{10}-49 = (a^{5}+7)(a^{5}-7)$

} notice if you have an even exponent, you have a perfect square.

④ $x^{4}-16 = (x^{2}+4)(x^{2}-4) = (x^{2}+4)(x+2)(x-2)$

Sum of squares is always a prime factor, which means that there is no more factoring needed (possible).

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# (4) Sum and Difference of Cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$

(square: $a \to a^2$, $b \to b^2$; same; opposite; always positive)

$a^3-b^3=(a-b)(a^2+ab+b^2)$

S O AP

e.g. $x^3+8=(x+2)(x^2-2x+4)$

$\sqrt[3]{x^3}=x$, $\sqrt[3]{8}=2$

$x^3-8=(x-2)(x^2+2x+4)$

$1^3=1$
$2^3=8$
$3^3=27$
$4^3=64$
$5^3=125$
⋮

Ex ① $8x^3-27$

$\sqrt[3]{8x^3}=\sqrt[3]{8}\sqrt[3]{x^3}=2x$. $\sqrt[3]{27}=3$.

$=(2x-3)((2x)^2+(2x)(3)+3^2)$

$=(2x-3)(4x^2+6x+9)$

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## Hawkes Practice.

(1) $-8x^{\frac{7}{3}}-14x$

NOT a polynomial because $\frac{7}{3}$ is a fraction

(2) $x^5y^6z$.

It is a polynomial because every exponent is whole numb

degree is $5+6+1=12$.

It is a monomial because no plus/minus sign separating terms.

(3) $(-6x^4+3-5x^3)-(12-7x^4)$

$=-6x^4+3-5x^3-12+7x^4$

$=(-6+7)x^4+(3-12)-5x^3=x^4-5x^3-9$

Combine like terms:
1) same base
2) same exponent

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(4) Factor the following binomial if possible

$125x^6 + 64y^3$

$= (5x^2+4y)((5x^2)^2 - (5x^2)(4y) + (4y)^2)$

$= (5x^2+4y)(25x^4 - 20x^2y + 16y^2)$

$\sqrt[3]{125x^6} = \sqrt[3]{125}\,\sqrt[3]{x^6} = 5x^2$

$\sqrt[3]{64y^3} = \sqrt[3]{64}\,\sqrt[3]{y^3} = 4y$