Techniques in Factoring Polynomials

1) Always factor out the largest common factor first.

E.g., \(6n^3 + 24n^2 + 15n\). < this is a trinomial

First look at the coefficients.

3 is a common factor among \((6, 24, 15)\)

Then look at the variables.

\(n\) is a factor among \((n^3, n^2, n)\). called a monomial.

\(\Rightarrow \ 6n^3 + 24n^2 + 15n = 3n(2n^2 + 8n + 5)\)

Definition:

The degree is the highest exponent on the variable.

If there are more than one variable, the degree is defined to be the largest sum of exponent out of all terms.

E.g. the degree of the binomial \(3x^2y^3 - 4xy^2\) is \(2 + 3 = 5\)

\(\Delta\)

The sum here is 3, which is less than 5.

the degree of the trinomial \(2xy + 3x - 5y\) is \(1 + 1 = \textit{not}\)

Distributive Property.

Multiplying a monomial with a polynomial uses the distributive property.

Multiplying things of the same base, you add the exponent.

E.g. \(2x(5x^2 - 6x + 7) = (2x)(5x^2) - (2x)(6x) + (2x)(7) = 10x^3 - 12x^2 + 14\)

Factoring out the largest common factor is undoing the distribution.

\(10x^3 - 12x^2 + 14x + 5x^2 - 5x + 7\)
1. When you are asked to factor something, that means that you need to write the original expression into a product (multiplication) of factors.

Recall: \(2 \cdot 5 = 10\) means \(2\) and \(5\) are factors of \(10\). Thus, \(10\) is the product of its factors.

Typically, we are interested in prime factorization.

\[\text{Ex} \text{ } 2 \quad a(b-2) + c(b-2) = (b-2)(a+c)\]

\[\text{Ex} \text{ } 3 \quad 18a^2b - 15ab^2\]

- With coefficients: \(18\) and \(15\), \(\gcd(18,15) = 3\)
- With \(a\) terms: a common factor is \(a\)
- With \(b\) terms: \(b\) and \(b^2\), a common factor is \(b\)

\[18a^2b - 15ab^2 = 3ab(6a - 5b)\]

(2) Factoring by grouping.

- The polynomial has 4 terms, and it might call for factoring by grouping.

- Procedure:
  1. Group terms into pairs, which have things in common.
  2. Factor out the largest common factors from each pair.
  3. Factor out the common binomial.
Ex 1  \( x^3 + 3x^2 + 6x + 18 \)
\[ = (x^3 + 3x^2) + (6x + 18) \]
\[ = x^2(x + 3) + 6(x + 3) \]
\[ = (x + 3)(x^2 + 6) \]

Ex 2  \( 2x^3 - 3x^2 + 6x - 9 \)
\[ = (2x^3 - 3x^2) + (6x - 9) \]
\[ = x^2(2x - 3) + 3(x - 3) \]
\[ = (2x - 3)(x^2 + 3) \]

These two products are equivalent because of the commutative property of multiplication. \( ab = ba \)

Addition also has commutative property. \( a + b = b + a \)

Ex 3  \( a^3 - 3a^2 - 2a + 6 \)
\[ = (a^3 - 3a^2) + (-2a + 6) \]
\[ = a^2(a - 3) + (-2)(a - 3) \]
\[ = (a^2 - 2)(a - 3) \]

Ex 3  \( a^3 - 3a^2 - 2a + 6 \)
\[ = (a^3 - 2a) + (-3a^2 + 6) \]
\[ = a(a - 2) + (-3)(a^2 - 2) \]
\[ = (a - 2)(a - 3) \]

(3) Difference of two Squares.

Recall Eq. 1.1 \( (x + 5)(x - 5) = x^2 - 5x + 5x - 25 = x^2 - 25 \)

This yields a difference of squares.

To reverse this process: \( x^2 - 25 = (x + 5)(x - 5) \)
Ex 1  \( m^2 - 4 \)  
\[ = (m + 2)(m - 2) \]
\[ \sqrt{m^2} = m \]
\[ \sqrt{4} = 2 \]

Ex 2  \( 16x^2 - 9 \)
\[ = (4x + 3)(4x - 3) \]
\[ \sqrt{16x^2} = 4x \]
\[ \sqrt{9} = 3 \]

Ex 3  \( 4x^2y^2 - 4xz^2 \)
\[ = 4x(y^2 - z^2) \]
\[ = 4x(y + z)(y - z) \]

Notice \((y^2 - z^2)\) is a difference of two squares.

Ex 4  \( 16x^2 - 25z^2 \)
\[ = (4x + 5z)(4x - 5z) \]
\[ \sqrt{16x^2} = 4x \]
\[ \sqrt{25z^2} = 5z \]