

Factoring Greatest common factors out and factoring by grouping

Multiplying polynomials

Ex. $2x(x+3) = 2x^2 + 6x$

Factor
$$2x^2 + 6x = 2x \left(\frac{2x^2}{2x} + \frac{6x}{2x} \right)$$
$$= 2x(x+3)$$

Factoring tips

- 1st rule in factoring - always check for a GCF
- when you have a variable in common, always factor out the variable raised to the smallest power.
- check multiplying back
- Factor out the GCF

$$7x^3 + 14x^2 - 21x$$
$$= 7x(x^2 + 14x - 21)$$

this might factor further

$$9a^4b - 18a^5b + 27a^6b$$
$$= 9a^4b(1 - 2a + 3a^2)$$
$$= 9a^4b(3a^2 - 2a + 1)$$

this might factor further

- Then see if the factors will factor any further.
How many terms? 4 terms - try to factor by grouping.

Factor out GCF

leading coefficient

$$-3x^2 + 6x - 33$$

$$= -3(x^2 - 2x + 11)$$

OR

$$= 3(-x^2 + 2x - 11)$$

Factor out the GCF

$$2w(x+3) - 5(x+3)$$

$$(x+3)(2w-5)$$

Factor out the GCF

$$8y(a+b) + 9(a+b)$$

$$= (a+b)(8y+9)$$

Factor

$$3ax + 12a + 2bx + 8b$$

$$= 3a(x+4) + 2b(x+4)$$

$$= (x+4)(3a+2b)$$

check:

$$(x+4)(3a+2b)$$

$$= 3ax + 2bx + 12a + 8b$$

- Steps to factoring by grouping - look for GCF first.

① collect the terms into groups so that each group has a common factor.

② factor out common factor from each group if there is one, otherwise, try to rearrange the terms.

③ Now if there is a common factor, factor it out.

Factor

$$\begin{aligned}
 & (xy - xz) + (7y - 7z) \\
 &= x(y - z) + 7(y - z) \\
 &= (y - z)(x + 7)
 \end{aligned}$$

$$\begin{aligned}
 & b^4 + b^3 - 4b - 4 \\
 &= (b^4 + b^3) + (-4b - 4) \\
 &= b^3(b + 1) - 4(b + 1) \\
 &= (b + 1)(b^3 - 4)
 \end{aligned}$$

Factor

$$ay + bx + by + ax$$

Rearrange this

$$\begin{aligned}
 &= (ay + by) + (bx + ax) \\
 &= y(a + b) + x(b + a) \\
 &= y(a + b) + x(a + b) \\
 &= (a + b)(y + x)
 \end{aligned}$$

Factor

$$\begin{aligned}
 & 15x^4 + 15x^2y^2 + 10x^3y + 10xy^3 \\
 &= 5x \left[(3x^3 + 3xy^2) + (2x^2y + 2y^3) \right] \\
 &= 5x \left[3x(x^2 + y^2) + 2y(x^2 + y^2) \right] \\
 &= 5x(x^2 + y^2)(3x + 2y)
 \end{aligned}$$

- 3 terms $x^2 + bx + c$

looking coefficient of 1

factoring trinomials in the form

$$x^2 + bx + c$$

Multiply

$$(x+5)(x-3)$$

$$= x^2 - 3x + 5x - 15$$

$$= x^2 + 2x - 15$$

$$= x^2 + bx + c$$

$$(x-3)(x+5)$$

Factor

$$x^2 + 2x - 15$$

	<u>$c = -15$</u>
	$1(-15)$
	$-1(15)$
	$3(-5)$
	$-3(5)$

- ① list all pairs whose product is "c"
- ② Now find the one pair whose sum is b

$$(x \square)(x \square)$$

Factor

$$x^2 + 10x + 21$$

$$= (x+3)(x+7)$$

Add
$b = 10$
<hr/>
$3 + 7$

Mult
$c = 21$
<hr/>
$1 \cdot 21$

$3 + 7$	$\leftarrow 3 \cdot 7$
	$(-1)(-21)$
	$(-3)(-7)$

Check by FOIL

$$(x+3)(x+7)$$

$$= x^2 + 7x + 3x + 21$$

$$= x^2 + 10x + 21$$

Factor

$$x^2 + 4x - 45$$

$$= (x+9)(x-5)$$

OR

$$(x-5)(x+9)$$

Factor

$$x^2 - 5x - 14$$

$$= (x-7)(x+2)$$

Factor

$$-a^2 + 6a - 8$$

$$= -(a^2 - 6a + 8)$$

$$= -(a-4)(a-2)$$

check:

$$-(\overbrace{a-4}^{\rightarrow})(\overbrace{a-2}^{\rightarrow})$$

$$= -(a^2 - 2a - 4a + 8)$$

$$= -(a^2 - 6a + 8)$$

$$= -a^2 + 6a - 8$$

$$-3a^2 + 15ab - 12b^2$$

$$= -3(a^2 - 5ab + 4b^2)$$

$$= -3(a-4b)(a-b)$$