Factoring

Greatest common factors out and factoring by grouping

Multiplying polynomials

Ex. \[2x(x+3) = 2x^2 + 6x\]

Factor

\[2x^2 + 6x = 2x\left(\frac{2x}{2x} + \frac{6x}{2x}\right)\]

\[= 2x(x+3)\]

Factoring tips

- 1st rule in factoring — always check for a GCF
- When you have a variable in common, always factor out the variable raised to the smallest power.
- Check multiplying back
- Factor out the GCF

\[7x^3 + 14x^2 - 21x\]

\[= 7x(x^2 + 14x - 21)\]

this might factor further

- Then see if the factors will factor any further.
- How many terms? 4 terms — try to factor by grouping.

\[9a^4b - 18a^5b + 27a^6b\]

\[= 9a^4b (1 - 2a + 3a^2)\]

\[= 9a^4b (3a^2 - 2a + 1)\]

this might factor further
Factor out GCF

\[-3x^2 + 6x - 33\]

\[= -3(x^2 - 2x + 11)\]

or

\[= 3(-x^2 + 2x - 11)\]

Factor out the GCF

\[2W(x+3) - 5(x+3)\]

\[(x+3)(2W-5)\]

Factor out the GCF

\[8y(a+b) + 9(a+b)\]

\[= (a+b)(8y+9)\]

Factor

\[3ax + 12a + 2bx + 8b\]

\[= 3a(x+4) + 2b(x+4)\]

\[= (x+4)(3a+2b)\]

Check:

\[(x+4)(\frac{3}{2}a+2b)\]

\[= 3ax + 2bx + 12a + 8b\]

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Steps to factoring by grouping - look for GCF first.

1. Collect the terms into groups so that each group has a common factor.
2. Factor out common factor from each group if there is one. Otherwise, try to rearrange the terms.
3. Now if there is a common factor, factor it out.
\[
\frac{\text{Factor}}{(xy - x^2) + (7y - 7z)}
\]
\[
= x(y - x) + 7(y - z)
\]
\[
= (y - x)(x + 7)
\]

\[
4^4 + 3^3 - 4b - 4
\]
\[
= (4^4 + 3^3) + (-4b - 4)
\]
\[
= 5^3(b^1) - 4(b + 1)
\]
\[
= (b + 1)(5^3 - 4)
\]

\[
\text{Factor}
\]
\[
ay + by + bx + ax
\]
\[
\text{Rearrange this}
\]
\[
= (ay + by) + (bx + ax)
\]
\[
= y(a + b) + x(b + a)
\]
\[
= y(a + b) + x(a + b)
\]
\[
= (a + b)(y + x)
\]

\[
\text{Factor}
\]
\[
15x^4 + 15x^2y + 10xy^2 + 10y^3
\]
\[
= 5x\left[(3x^3 + 3xy^2) + (2xy + 2y^3)\right]
\]
\[
= 5x\left[3x(x^2 + y^2) + 2y(x^2 + y^2)\right]
\]
\[
= 5x(x^2 + y^2)(3x + 2y)
\]
- 3 terms \( x^2 + bx + c \)

looking coefficient of 1

Factoring trinomials in the form \( x^2 + bx + c \)

\[
\begin{align*}
\text{Multiply} & \quad \frac{c}{x^2 + 2x - 15} \\
(x+5)(x-3) & \quad = x^2 - 3x + 5x - 15 \\
& \quad = x^2 + 2x - 15 \\
= x^2 + bx + c & \quad (x-3)(x+5) \\
\end{align*}
\]

1. list all pairs whose product is \( c \)
2. now find the one pair whose sum is \( b \)

\[
(x \Box) (x \Box)
\]

\[
\begin{align*}
\text{Factor} & \quad x^2 + 10x + 21 \\
= (x + 3)(x + 7) & \quad \text{add} \quad b = 10 \\
\text{Check by FOIL} & \quad \text{Mult} \quad c = 21 \\
(x+3)(x+7) & \quad \frac{c}{1, 21} \\
= x^2 + 7x + 3x + 21 & \quad 3+7 \leftarrow 3, 7 \quad (-11)(-21) \\
= x^2 + 10x + 21 & \quad (-3)(-7)
\end{align*}
\]
\[
\frac{\text{Factor}}{x^2 + 4x - 45} = (x+9)(x-5)
\]

or
\[
(x-5)(x+9)
\]

\[
\text{Factor} \quad x^2 - 5x - 14 = (x-7)(x+2)
\]

\[
\text{Factor} \quad -a^2 + 6a - 8 = -(a-2)(a-4)
\]

Check:
\[
-(a-2)(a-4) = -(a^2 - 2a - 4a + 8)
= -a^2 + 6a - 8
\]

\[
-3a^2 + 15ab - 12b^2 = -3(a^2 - 5ab + 4b^2)
= -3(a - 4b)(a - b)
\]