Abstract: A graph is a unit-distance graph if it can be drawn in the plane so that the distance between any pair of adjacent vertices is exactly one. A unit distance graph is maximal if the addition of any edge results in a graph which is not a unit-distance graph. Let \( u(n) \) and \( U(n) \) denote the minimum and maximum number of edges in a maximal unit-distance graph of order \( n \). Finding a formula for \( u(n) \) and \( U(n) \) seems difficult. Some asymptotic results have been proven by Erdős (1946) and Spencer, Szemerédi and Trotter (1984), but still very little is known even for small \( n \). The exact values are known for \( n \leq 7 \). We show through proofs and computation (i.e., graph theory, algebraic geometry, combinatorics, optimization, etc.) that \( u(8) = 12, U(8) = 14 \), and we determine all maximal unit distance graphs of order eight. (This is joint work with Jonathan Berry, Bob Carr, Jill Cochran, Cynthia A. Phillips and Brigitte Servatius.)

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