Abstract: In complex analysis, analytic functions have several very nice properties that real-valued functions do not have. For over 100 years, mathematicians have studied analytic functions and their properties. In 1984, mathematicians began investigating a generalization of analytic functions known as complex-valued harmonic mappings. We will introduce this new family of complex-valued harmonic mappings and discuss some surprising differences between them and analytic functions. Then we will show how these complex-valued harmonic mappings are connected with differential geometry, a different area of mathematics. Specifically, complex-valued harmonic mappings can be lifted from the complex plane into $\mathbb{R}^3$ forming minimal surfaces which can be modeled as soap films. After briefly presenting some background in differential geometry and minimal surface theory, we will discuss a new result that uses the convex combinations of harmonic mappings to construct minimal surfaces over nonconvex domains. This provides a new way to construct the family of Jenkins-Serrin minimal surfaces.

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